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Bound-Preserving Operators and Bernstein Type Inequalities

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Abstract. Let \mathbb{D} be the unit disk in the complex plane \mathbb{C} and

$$\|p\| := \max_{z \in \partial\mathbb{D}} |p(z)|,$$

where $p(z) = \sum_{k=0}^n a_k(p)z^k$ is a polynomial of degree at most n and $a_k(p) \in \mathbb{C}$. The following sharpening of Bernstein's inequality

$$\|p'\| + \frac{2n}{n+2}|a_0(p)| \leq n\|p\|$$

has been proved by Ruscheweyh. Our main contribution concerns the case of equality which has remained unsolved since 1982. We prove another inequality of Bernstein type that leads to an improvement of the upper bound for $\|p'\|$ under some additional condition.

Keywords. Polynomials, bound-preserving operators, Bernstein type inequalities.

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