
Dimiter P. Dryanov, Mohammed A. Qazi, and Qazi I. Rahman

Local Behavior of Entire Functions of Exponential Type

CMFT 2 No.2 (2002), 319–336. [ISSN 1617-9447]

Abstract. We consider entire functions of exponential type $\tau > 0$, whose modulus is bounded by a constant M at the extrema of $\sin(\tau z)$, and which vanish at the origin. Extending a result of L. Hörmander, we show that if f is any such function, then $|f(x)| \leq M|\sin(\tau x)|$ for all $x \in (-\pi/(2\tau), \pi/(2\tau))$, provided that $f(x) = o(x)$ as $x \rightarrow \pm\infty$; furthermore, equality holds at any point $x \in (-\pi/(2\tau), 0) \cup (0, \pi/(2\tau))$ if and only if $f(z) \equiv e^{i\gamma} \sin(\tau z)$ for some $\gamma \in \mathbb{R}$. This also generalizes a result due to R. P. Boas Jr. about trigonometric polynomials. Besides, we prove some other results for entire functions of order 1 and type $\tau > 0$, one being an analogue of a result of M. Riesz about trigonometric polynomials whose zeros are all real and simple.

Keywords. Entire functions, exponential type, local behaviour.

2000 MSC. 30A10, 30C15, 30D10, 30D15.

Received. March 5, 2003.