
Gunter Semmler and Elias Wegert

Separation Principles and Riemann-Hilbert Problems

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Abstract. Restrictions imposed on the boundary values of holomorphic functions induce restrictions on their values at interior points. The paper is devoted to the following related question:

Let A be a subclass of the Hardy space H^1 in the complex unit disk \mathbb{D} and for each $t \in \partial\mathbb{D}$ let the complex plane be divided into an upper and a lower domain by some curve M_t . If we know that the boundary values of holomorphic functions $w_+ = u_+ + iv_+$ and $w_- = u_- + iv_-$ in A lie in the upper and the lower domain respectively, what can we conclude about the relative position of $w_+(0)$ and $w_-(0)$?

The problem is studied for several classes A and with different assumptions on the separating curves M_t . A number of counterexamples illustrates the limitations of the results.

One main tool in the investigations is a non-linear boundary value problem of Riemann-Hilbert type, which is also of independent interest. Using an approximation procedure and an argument based on normal families we extend earlier results on existence and uniqueness of solutions for smooth problems to the case of piecewise continuous boundary conditions.

Keywords. Holomorphic functions, normal family, separation principle, Riemann-Hilbert problem.

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