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Asymptotic Behaviour of the Conformal Representation of a Jordan Domain with a Small Hole in Schauder Spaces

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Abstract. We consider a suitably normalized Riemann map $g[\zeta]$ of the plane annulus $\mathbb{A}(r[\zeta], 1) \equiv \{z \in \mathbb{C} : r[\zeta] < |z| < 1\}$ to the plane annular domain $\mathbb{A}[\zeta]$ enclosed by the pair of Jordan curves $\zeta \equiv (\zeta^i, \zeta^o)$. Here ζ^i is of the form $w + \epsilon\xi$, where w is a point in the domain enclosed by the external curve ζ^o , and ξ is a curve enclosing 0, and $\epsilon > 0$ is a real parameter. We analyze the behaviour of the corresponding $g[\zeta]$ as ϵ tends to 0. More precisely, we show that the nonlinear operator which takes the quadruple $(w, \epsilon, \xi, \zeta^o)$ to the corresponding triple of functions

$$\left(r^{-1}[\zeta]g[\zeta]^{(-1)} \circ \zeta^i, g[\zeta]^{(-1)} \circ \zeta^o, \epsilon^{-1}r[\zeta] \right)$$

can be continued real analytically around a singular quadruple $(w, 0, \xi, \zeta^o)$ corresponding to an annular domain with an interior degenerate curve. As a corollary, one can deduce information on the behaviour of the relative capacity of the domain enclosed by $\zeta^i = w + \epsilon\xi$ with respect to that enclosed by ζ^o as ϵ tends to 0.

Keywords. Conformal representation, perforated domains, nonlinear operators, singular perturbation, asymptotic behaviour.

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