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**Taylor Coefficients of Negative Powers of Schlicht Functions**

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**Abstract.** Let $\mathcal{S}$ denote the class of normalized schlicht functions in the unit disk. We consider for $f \in \mathcal{S}$ and $\lambda < 0$ the Taylor coefficients $a_n(\lambda, f)$ of $(f(z)/z)^\lambda$ and prove that $|a_n(\lambda, f)| \leq |a_n(\lambda, k)|$ for every $f \in \mathcal{S}$ and every $1 \leq n \leq -\lambda + 1$, where $k(z) = z(1-z)^{-2}$ is the Koebe function. We also give a necessary condition such that the Koebe function maximizes the functional

$$\sum_{k=1}^{n} \sigma_k |a_k(\lambda, f)|^2$$

in the class $\mathcal{S}$ for given weights $\sigma_k \in \mathbb{R}$. These results supplement and complement previous results due to de Branges, Hayman and Hummel and others. Our proofs are based on the Löwner differential equation combined with optimal control methods.

**Keywords.** Taylor coefficients, univalent functions, Löwner's method, optimization.

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