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Asymptotics for Minimal Blaschke Products and Best $\mathbf{L_1}$ Meromorphic Approximants of Markov Functions

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Abstract. Let μ be a positive Borel measure with support $\operatorname{supp}\mu = E \subset (-1, 1)$ and let

$$\Delta_n = \inf_{B \in \mathcal{B}_n} \int_E |B(x)|^2 d\mu(x),$$

where $\mathcal{B}_n$ is the collection of all Blaschke products of degree n. Denote by $B_n \in \mathcal{B}_n$ a Blaschke product that attains the value Δ_n. We investigate the asymptotic behavior, as $n \to \infty$, of the minimal Blaschke products B_n in the case when the measure μ with support $E = [a, b]$ satisfies the Szegő condition:

$$\int_a^b \frac{\log(d\mu/dx)}{\sqrt{(x-a)(b-x)}}\, dx > -\infty.$$

At the same time, we shall obtain results related to the convergence of best L_1 approximants on the unit circle to the Markov function

$$f(z) = \frac{1}{2\pi i} \int_E \frac{d\mu(x)}{z - x}$$

by meromorphic functions of the form P/Q, where P belongs to the Hardy space H_1 of the unit disk and Q is a polynomial of degree at most n. We also include in an appendix a detailed treatment of a factorization theorem for Hardy spaces of the slit disk, which may be of independent interest.

Keywords. Blaschke products, meromorphic approximation, Markov functions, best approximation.

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