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Riemann-Hilbert Analysis for Laguerre Polynomials with Large Negative Parameter

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Abstract. In this paper we study the asymptotic behavior of Laguerre polynomials $L_n^{(\alpha_n)}(nz)$ as $n \rightarrow \infty$, where α_n is a sequence of negative parameters such that $-\alpha_n/n$ tends to a limit $A > 1$ as $n \rightarrow \infty$. These polynomials satisfy a non-hermitian orthogonality on certain contours in the complex plane. This fact allows the formulation of a Riemann-Hilbert problem whose solution is given in terms of these Laguerre polynomials. The asymptotic analysis of the Riemann-Hilbert problem is carried out by the steepest descent method of Deift and Zhou, in the same spirit as done by Deift et al. for the case of orthogonal polynomials on the real line. A main feature of the present paper is the choice of the correct contour.

Keywords. Riemann-Hilbert problems, generalized Laguerre polynomials, strong asymptotics, steepest descent method.

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