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A Note on the Hypergeometric Mean Value

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Abstract. Recent efforts to obtain bounds for the complete elliptic integral

$$\frac{\pi}{2} \cdot {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; 1; r^2\right)$$

in terms of power means and other related means have precipitated the search for similar bounds for the more general ${}_2F_1(\alpha, \beta; \gamma; r)$. In an early paper, B. C. Carlson considered the approximation of the *hypergeometric mean values* $({}_2F_1(-a, b; b + c; r))^{1/a}$ in terms of *means of order t* , given by $M_t(s, r) := \{(1 - s) + s(1 - r)^t\}^{1/t}$. In this note, a refinement of one such approximation is established by first proving a general positivity result involving ${}_3F_2$.

Keywords. Hypergeometric function, generalized hypergeometric function, means of order t .

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