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Connectedness of Julia Sets of Rational Functions

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Abstract. For a polynomial P it is well known that its Julia set \mathcal{J}_P is connected if and only if the orbits of the finite critical points are bounded. But there is no such simple criteria for the connectedness of the Julia set of a rational function. Indeed, up to the very nice result of Shishikura that any rational function which has one repelling fixed point only has a connected Julia set almost nothing is known on the connectivity. In the first part of the paper we give constructive sufficient conditions for a basin of attraction to be completely invariant and the Julia set to be connected. Then it is shown that the connectedness of a basin of attraction depends heavily on the fact whether the critical points from the basin tend to the attracting fixed point z_0 via a preimage of z_0 or not. As a consequence we obtain for instance that rational functions with a finite postcritical set or with a Fatou set which contains no Herman rings and each component of which contains at most one critical point, counted without multiplicity, have a connected Julia set.

Keywords. Julia set, Fatou set, basin of attraction, connectedness.

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