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Singularities in Baker Domains

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Abstract. Let U be a Baker domain of a transcendental entire function f. Denote by λ_U the hyperbolic metric in U and, for $w \in U$ and $n \in \mathbb{N}$, define $\rho_n(w) = \lambda_U(f^{n+1}(w), f^n(w))$ and $\rho(w) = \lim_{n\to\infty} \rho_n(w)$. Here f^n denotes the n–th iterate of f. It is shown that if the set of singularities of f^{-1} that are contained in U is bounded, then

$$\rho_n(w) = \frac{1}{2n} + a\frac{\log n}{n^2} + \mathcal{O}\left(\frac{1}{n^2}\right)$$

for some $a \in \mathbb{R}$ if $\rho(w) = 0$ and

$$\rho_n(w) = \rho(w) + \frac{b}{n^3} + \mathcal{O}\left(\frac{1}{n^4}\right)$$

for some $b \geq 0$ if $\rho(w) > 0$, but $\inf_{w\in U} \rho(w) = 0$. The result is applied to certain entire functions of finite order.

Keywords. Iteration, entire function, inner function, Julia set, Baker domain, singularity, residue fixed point index, Denjoy-Wolff point, hyperbolic metric.

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