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**The Wronski map and Grassmannians of Real Codimension 2 Subspaces**

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**Abstract.** We study the map which sends a pair of real polynomials  $(f_0, f_1)$  into their Wronski determinant  $W(f_0, f_1)$ . This map is closely related to a linear projection from a Grassmannian  $G_{\mathbb{R}}(m, m + 2)$  to the real projective space  $\mathbb{R}P^{2m}$ . We show that the degree of this projection is  $\pm u((m + 1)/2)$  where  $u$  is the  $m$ -th Catalan number. One application of this result is to the problem of describing all real rational functions of given degree  $m + 1$  with prescribed  $2m$  critical points. A related question of control theory is also discussed.

**Keywords.** Rational functions, Grassmann varieties, enumerative geometry, control of linear systems.

**2000 MSC.** 26C15, 14N10, 14N15, 14M15, 93C05, 93C35.

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