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N. Khamsemanan, R. Connelly

Two-distance Preserving Functions

This paper provides a proof the following result: Suppose $n \geq 2$, $0 < c < s$, $0 < c/s < \frac{\sqrt{5}-1}{2}$, and $f: E^n \rightarrow E^m$ is any function such that for all $p, q \in E^n$, the following two properties hold: (1) If $|p - q| = c$, then $|f(p) - f(q)| \leq c$. (2) If $|p - q| = s$, then $|f(p) - f(q)| \geq s$. Then f is congruence. This result originally proved Bezdek and Connelly [BC]. However Bezdek and Connelly's result depends on a result of Rado et al. [R], where the similar result holds but for $\frac{1}{\sqrt{3}}$ replacing $\frac{\sqrt{5}-1}{2}$. Rado's proof is long and time consuming to verify.

In this paper, we provide our own separate proof, using the mathematical software, Maple, which is independent of Rado's proof. Our result is shorter, more systematic and, we believe, easier to verify.

[BC] Bezdek, K.; Connelly, R.: Two-Distance Preserving Functions From Euclidean Space, *Period. Math. Hungar.* 39(1-3) (1999) 185-200.

[R] Rado, F.; Andreescu, D.; Valcan, C.: Mapping of E^n to E^m preserving two distance, *Seminar on Geometry (Cluj-Napoca)* 9-22, 1986.